

2641/01 Statistics 1

June 2004

Mark Scheme

<p>1 (i)</p>	<table border="1" data-bbox="300 192 799 371"> <thead> <tr> <th>Aspect</th> <th>A</th> <th>B</th> <th>C</th> <th>D</th> <th>E</th> </tr> </thead> <tbody> <tr> <td>Person 1</td> <td>71</td> <td>63</td> <td>84</td> <td>57</td> <td>64</td> </tr> <tr> <td>Person 2</td> <td>12</td> <td>62</td> <td>20</td> <td>85</td> <td>31</td> </tr> <tr> <td>Rank 1</td> <td>2</td> <td>4</td> <td>1</td> <td>5</td> <td>3</td> </tr> <tr> <td>Rank 2</td> <td>5</td> <td>2</td> <td>4</td> <td>1</td> <td>3</td> </tr> <tr> <td>d</td> <td>-3</td> <td>2</td> <td>-3</td> <td>4</td> <td>0</td> </tr> </tbody> </table> <p data-bbox="312 371 676 405">$\Sigma d^2 = 9 + 4 + 9 + 16 + 0 = 38$</p> <p data-bbox="312 439 667 472">Spearman's rank correlation</p> <p data-bbox="312 506 708 562">Coefficient = $1 - \frac{6 \times 38}{5 \times 24} = \frac{-9}{10} = -0.9$</p>	Aspect	A	B	C	D	E	Person 1	71	63	84	57	64	Person 2	12	62	20	85	31	Rank 1	2	4	1	5	3	Rank 2	5	2	4	1	3	d	-3	2	-3	4	0	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>4</p>	<p>Correct ranks (or reverse)</p> <p>Attempt to find d (or d^2) from ranked or ordered data</p> <p>Correct formula for Spearman <i>used</i> and $r < 1$</p> <p>Correct answer -0.9 or $\frac{-9k}{10k}$ cao</p>
Aspect	A	B	C	D	E																																		
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d	-3	2	-3	4	0																																		
<p>(ii)</p>	<p>Spearman's rank correlation coefficient shows that the two people have different, opposite views, or no or little agreement when considering aspects of their job</p>	<p>B1</p> <p>1</p>	<p>Comment in context, consistent with r_s value $r < 1$</p>																																				
<p>2 (i)</p> <p>(ii)</p>	<p>Number of possible arrangements = $\frac{5!}{2} = \mathbf{60}$</p> <p>Number of arrangements in which the white bricks are at each end = 3! or Number of arrangements in which both bricks are at either end = $3! \times 2!$</p> <p>Therefore P(white bricks are at each end) $= \frac{3!}{60} = \frac{6}{60} = \mathbf{0.1}$</p> <p>or P(both white bricks at either end) $= \frac{3! \times 2!}{60} = \mathbf{0.2}$</p> <p>or P(white at each end or both at either end) = $0.1 + 0.2 = \mathbf{0.3}$</p>	<p>M1</p> <p>A1</p> <p>2</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>3</p>	<p>5! or 120 seen (not in 5C_3)</p> <p>60, cao</p> <p>3! Seen for either case</p> <p><i>their</i> 3! Divided by <i>their</i> (i)</p> <p>0.1 or 0.2 or 0.3 $\frac{k}{10k}$ or $\frac{k}{5k}$ or $\frac{3k}{10k}$</p>																																				

<p>3 (i)</p>	<p>Let X = number of heads in 4 randomly chosen spins of the coin. $P(W = -5) = P(X = 0 \text{ or } 1) = \frac{1}{16} + \frac{4}{16}$ $= \frac{5}{16}$ AG</p>	<p>M1 A1</p>	<p>Attempt to find $P(X = 0)$ or $P(X = 1)$ Wholly correct and convincing attempt (allow decimals)</p>																				
<p>(ii)</p>	<p>Distribution table for X</p> <table border="1" data-bbox="308 539 874 629"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>$P(X = x)$</td> <td>$\frac{1}{16}$</td> <td>$\frac{4}{16}$</td> <td>$\frac{6}{16}$</td> <td>$\frac{4}{16}$</td> <td>$\frac{1}{16}$</td> </tr> </table> <p>Distribution table for W</p> <table border="1" data-bbox="308 689 874 790"> <tr> <td>w</td> <td>-5</td> <td>5</td> <td>10</td> </tr> <tr> <td>$P(W = w)$</td> <td>$\frac{5}{16}$</td> <td>$\frac{6}{16}$</td> <td>$\frac{5}{16}$</td> </tr> </table>	x	0	1	2	3	4	$P(X = x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$	w	-5	5	10	$P(W = w)$	$\frac{5}{16}$	$\frac{6}{16}$	$\frac{5}{16}$	<p>M1 A1</p>	<p>A clear attempt to derive $P(W = 5)$ or $P(W = 10)$ Wholly correct table</p>
x	0	1	2	3	4																		
$P(X = x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$																		
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<p>(iii)</p>	<p>$E(W) = (-5) \times \frac{5}{16} + 5 \times \frac{6}{16} + 10 \times \frac{5}{16}$ $= \frac{-25+30+50}{16} = \frac{55}{16} (=3.4375)$ AG</p>	<p>M1 A1</p>	<p>Use of $\sum wp$ for <i>their</i> distribution table, at least 2 wp terms added Wholly correct method</p>																				
<p>(iv)</p>	<p>$E(W^2) = (-5)^2 \times \frac{5}{16} + 5^2 \times \frac{6}{16} + 10^2 \times \frac{5}{16}$ $= \frac{775}{16}$ So $\text{Var}(W) = E(W^2) - [E(W)]^2$ $= \left(\frac{775}{16}\right) - \left(\frac{55}{16}\right)^2$ $= \frac{9375}{256} = 36.62.. = 36.6$ (3 sf)</p>	<p>M1 M1 A1</p>	<p>Use of $\sum w^2 p$ for <i>their</i> distribution table, at least 2 $w^2 p$ terms added Subtracting (<i>their mean</i>)² $\frac{9375k}{256k}$ or a.r.t. 36.6</p>																				

<p>4 (i)</p> <p>Since x is a controlled variable, only the y on x line is appropriate</p> $S_{xy} = \Sigma xy - \frac{(\Sigma x)(\Sigma y)}{n} =$ $= 244260 - \frac{550 \times 3717}{10} = 39825$ $S_{xx} = \Sigma x^2 - \frac{(\Sigma x)^2}{n}$ $= 38500 - \frac{550^2}{10} = 8250$ $\bar{x} = 55, \bar{y} = 371.7$ $b = \frac{S_{xy}}{S_{xx}} = \frac{39825}{8250} = 4.82727\dots$ $a = \bar{y} - b\bar{x} = 371.7 - 4.82727\dots \times 55$ $= 106.2$ <p>Equation of line is</p> $y = 4.82727\dots x + 106.2$ $y = 4.83x + 106 \text{ (3 sf)}$ <p>Estimated value of $x = \frac{(220 - 106.2)}{4.82727\dots}$</p> $= 23.57438$ $= \mathbf{23.6 \text{ (3 sf)}}$		<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p>	<p>Use of y on x line</p> <p>$\frac{S_{xy}}{S_{xx}}$ used</p> <p>May be implied if calculator routine is used</p> <p>Using $= \bar{y} - b\bar{x}$ with <i>their</i> b</p> <p>$y = 4.83x + 106.2$, or correct equivalent (does not need to be in the form $y = a + bx$)</p> <p>Substitute $y = 220$ into <i>their</i> equation</p> <p>a.r.t. 23.6</p> <p>6</p> <p>Use of the x on y line</p> <p>For $\frac{S_{xy}}{S_{xx}}$ used</p> <p>Using $\bar{x} - b'\bar{y}$ with <i>their</i> b'</p> <p>$x = -21.1 + 0.205y$</p> <p>Substitute $y = 220$ into <i>their</i> equation</p>
<p>(ii)</p> $S_{yy} = \Sigma y^2 - \frac{(\Sigma y)^2}{n}$ $= 1576075 - \frac{3717^2}{10}$ $= 194466.1$ $r = \frac{S_{xy}}{\sqrt{S_{xx} \times S_{yy}}}$ $= \frac{39825}{\sqrt{8250 \times 194466.1}}$ $= 0.99427\dots = \mathbf{0.994 \text{ (3 sf)}}$ <p>This is a very high positive correlation so the estimate is likely to be reliable</p>		<p>M1</p> <p>A1</p> <p>B1</p>	<p>Calculator of formula correctly used or equivalent (may be implied)</p> <p>Correct answer, a.r.t. 0.994</p> <p>Comment consistent with <i>their</i> r value, provided $r < 1$</p> <p>3</p>

5 (i)	$X \sim B(10, \frac{1}{57})$ <p>Independence: whether Andy wins a particular lottery game is independent of whether he has won any other game. Two possible outcomes: for each game Andy either wins or loses.</p>	B1) B1) B1 3	Binomial stated $n = 10$ and $p = \frac{1}{57}$ stated clearly One valid comment in context
(ii)(a)	$P(X=2) = {}^{10}C_2 \times \left(\frac{1}{57}\right)^2 \times \left(\frac{56}{57}\right)^8$ $= 0.0120217633$ $= \mathbf{0.012}$	M1 M1 A1 3	Their ${}^nC_2 \times p^2 \times (1-p)^{n-2}$ used Wholly correct method a.r.t. 0.012
(b)	$P(X > 2)$ $= 1 - P(X=0) - P(X=1) - P(X=2)$ $= 1 - [0.83778\dots + 0.14960\dots + 0.01202\dots]$ $= 0.00059074\dots$ $= \mathbf{0.000591 (3 sf)}$	M1 M1 A1 3	$1 - [P(X=0)+P(X=1)+P(X=2)]$ with at least 2 probs attempted Wholly correct method a.r.t. 0.0006
(iii)	$E(X) = np = 10 \times \frac{1}{57} = \frac{10}{57}$ $= 0.175438\dots = \mathbf{0.175 (3 sf)}$	B1 1	$= \frac{10k}{57k}$ or a.r.t. 0.175

6 (i)	<table border="1"> <thead> <tr> <th>Mass, m in kg</th> <th>Cumulative frequency</th> </tr> </thead> <tbody> <tr> <td>$m < 5$</td> <td>2</td> </tr> <tr> <td>$m < 10$</td> <td>9</td> </tr> <tr> <td>$m < 15$</td> <td>26</td> </tr> <tr> <td>$m < 20$</td> <td>45</td> </tr> <tr> <td>$m < 30$</td> <td>53</td> </tr> <tr> <td>$m < 50$</td> <td>60</td> </tr> </tbody> </table>	Mass, m in kg	Cumulative frequency	$m < 5$	2	$m < 10$	9	$m < 15$	26	$m < 20$	45	$m < 30$	53	$m < 50$	60	M1	At least one correct cumulative frequency seen, other than 2
Mass, m in kg	Cumulative frequency																
$m < 5$	2																
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		M1 A1 3	At least 4 points correct with the correct (u.c.b., cum freq) Wholly correct diagram														
6 (ii)	<p>From the graph</p> <p>Reading from the CF axis at 15 (or 15.25) for Q_1, at 30 (or 30.5) for Q_2, at 45 (or 45.75) for Q_3</p>	M1 A1 A1 3	Correct method for <i>either</i> the median or for a quartile <i>Their</i> Q_2 from <i>their</i> CF curve, provided u.c.b.'s used <i>Their</i> IQR from <i>their</i> CF curve														
6 (iii)		M1 M1 A1 3	A recognisable attempt at a boxplot At least 4 from 4: <i>their</i> Q_1 : <i>their</i> Q_2 <i>their</i> Q_3 : 47 correctly plotted Wholly correct diagram														
6 (iv)	<p>Comment on skewness, range, IQR, 5 summary numbers, max and/or min values, symmetry</p>	B1 1	One valid feature of data which can be deduced more easily from a boxplot, but do not allow median and/or quartiles.														

7 (i)	Possible routes: $ABA \rightarrow \text{prob} = \frac{2}{3} \times \frac{3}{4}$ $ACA \rightarrow \text{prob} = \frac{1}{3} \times \frac{4}{5}$ $P(\text{back at A}) = \frac{1}{2} + \frac{4}{15} = \frac{15}{30} + \frac{8}{30}$ $= \frac{23}{30}$ AG	M1 M1 A1 3	One correct product seen Both correct routes identified (letters, probs, tree diagram) and one correct product. No other routes allowed. Wholly convincing and correct
7 (ii)	Possible routes= ABCA or ACBA So prob $= \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} + \frac{1}{3} \times \frac{1}{5} \times \frac{3}{4}$ $= \frac{2}{15} + \frac{1}{20} = \frac{8}{60} + \frac{3}{60}$ $= \frac{11}{60}$ or 0.183.. = 0.183 (3 sf)	M1 M1 M1 A1 4	One correct route identified Both correct routes identified and one correct product Wholly correct method (no other routes) $\frac{11k}{60k}$ or a.r.t. 0.183
7 (iii)	Possible routes $ACBCB \rightarrow \frac{1}{3} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{5}$ $ACBAB \rightarrow \frac{1}{3} \times \frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}$ $ACACB \rightarrow \frac{1}{3} \times \frac{4}{5} \times \frac{1}{3} \times \frac{1}{5}$ $ABACB \rightarrow \frac{2}{3} \times \frac{3}{4} \times \frac{1}{3} \times \frac{1}{5}$ $ABCAB \rightarrow \frac{2}{3} \times \frac{1}{4} \times \frac{4}{5} \times \frac{2}{3}$ $= \frac{1}{300} + \frac{1}{30} + \frac{4}{225} + \frac{1}{30} + \frac{4}{45} = \frac{53}{300} =$ $0.176666. = \mathbf{0.177}$ (3 s.f.)	M1 M1 M1 M1 A1 5	At least 4 correct routes chosen 2 correct routes identified and one correct 4-termed product 3 correct products all products correct and added (no other routes) $\frac{53k}{300k}$ or a.r.t. 0.177
7 (iii)	ALITER: (i) $\times \frac{1}{3} \times \frac{1}{5} +$ (ii) $\times \frac{2}{3} + \frac{1}{3} \times \frac{1}{5} \times \frac{1}{4} \times \frac{1}{5}$		